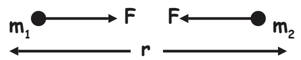
GRAVITATION

NEWTON'S LAW OF GRAVITATION



$$F = \frac{Gm_1m_2}{r^2}$$

G - Universal gravitational constant

Value of G

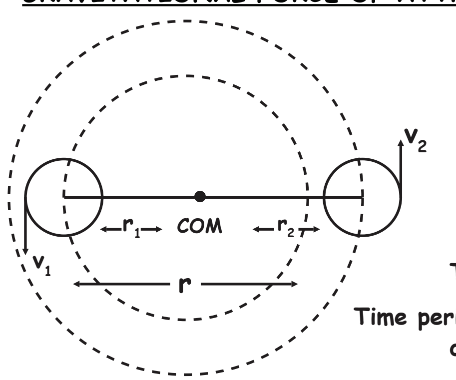
$6.67\times10^{-11}\ Nm^2Kg^{-2}$ (SI or MKS)

6.67×10^{-8} dyne cm^2g^{-2} (CGS)

Dimensional formula [G]

$M^{-1}L^3T^{-2}$

ROTATION OF 2 MASSES UNDER MUTUAL GRAVITATIONAL FORCE OF ATTRACTION



$$V_1 = \sqrt{\frac{Gm_2^2}{(m_1+m_2)\,r}}$$

$$V_2 = \sqrt{\frac{Gm_1^2}{(m_1+m_2)\,r}}$$

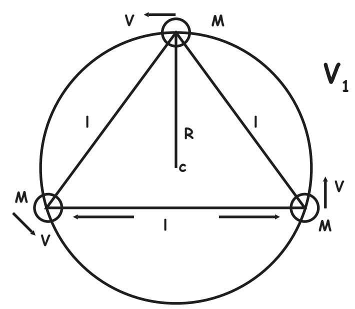
Time period $T^2 = 4\pi^2 \dfrac{r^3}{G(m_1+m_2)}$

or $\boxed{T^2 \propto r^3}$

IMPORTANT POINTS ABOUT GRAVITATIONAL FORCE

1. Gravitational force
* Always attractive in nature
* Independent of the nature of medium between masses
* Independent of presence or absence of other bodies
2. Are central forces, acts along the centre of gravity of two bodies.
3. Conservative force
4. Force between any two masses - Gravitational force
Force between earth and any other body - Force of gravity

THREE MASSES(EQUAL) REVOLVING UNDER MUTUAL GRAVITATIONAL FORCE

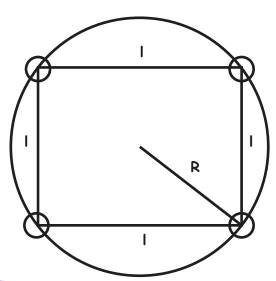


$$V_1 = \sqrt{\frac{GM}{l}} = \sqrt{\frac{GM}{R\sqrt{3}}}$$

Time period

$T^2 \propto l^3$

FOUR EQUAL MASSES UNDER MUTUAL GRAVITATIONAL FORCE

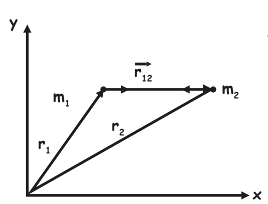


$$V = \sqrt{\frac{GM}{2\sqrt{2}\,l}(2\sqrt{2}+1)}$$

$$V = \frac{1}{2}\sqrt{\frac{GM}{R}(2\sqrt{2}+1)}$$

$T^2 \propto l^3$ Here $R = \dfrac{l}{\sqrt{2}}$

VECTOR FORM



$\vec{F}_{21}$ force on 2 due to 1

$\vec{r}_{12}$ vector from m_1 to m_2

$$\vec{F}_{21} = \frac{G\,m_1m_2}{r_{12}^2}(-\hat{r}_{12}) \quad \text{or} \quad -\frac{G\,m_1m_2}{r_{12}^3}\vec{r}_{12}$$

Similarly

$\vec{F}_{12}$ force on 1 due to 2

$$\vec{F}_{21} = \frac{G\,m_1m_2}{r_{12}^2}(\hat{r}_{12}) \quad \text{or} \quad \frac{G\,m_1m_2}{r_{12}^3}\vec{r}_{12}$$

Clearly

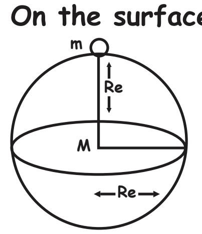
Newtons third law $\vec{F}_{21} = -\vec{F}_{12}$

Gravitational force is a two body interaction. Force between two particles does not depend on the presence or absence of other particles. The principle of superposition is valid here. "Force on a particle due to a no. of particles is the resultant of forces due to individual particles."

GRAVITY

Acceleration due to gravity

On the surface of earth $g = \dfrac{GM_e}{R_e^2}$



M - mass of earth

R - Radius of earth

[Put $GM_e = g\,R_e^2$ to solve problems easily]

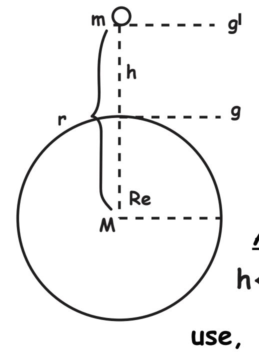
g IN TERMS OF DENSITY OF EARTH

$g = \frac{4}{3}\pi G\rho R_e \qquad g \propto \rho R_e$

"If density is mentioned use the above equation"

VARIATION IN THE VALUE OF ACCELERATION DUE TO GRAVITY

● Variation due to height 'h'



$$g^l = \frac{GM}{(R+h)^2}$$

$$g^l = \frac{gR^2}{(R+h)^2}$$

General Equation

Approximate equation

h<<<<<<R (h < 100 km)

use, $g^l = g\left(1 - \dfrac{2h}{R}\right)$

Note the point

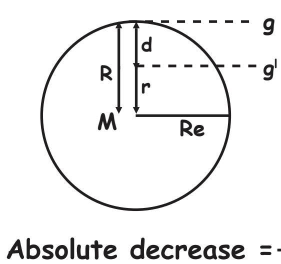
If h<<<<R, then decrease in the value of g with height

Absolute decrease $= \Delta g = g - g^l = \dfrac{2hg}{R}$

Fractional decrease $= \dfrac{\Delta g}{g} = \dfrac{g-g^l}{g} = \dfrac{2h}{R}$

Percentage decrease $= \dfrac{\Delta g}{g} = \dfrac{g-g^l}{g}\times 100 = \dfrac{2h}{R}\times 100$

● Variation due to depth 'd'



$$g^l = g\left[1 - \frac{d}{R}\right]$$

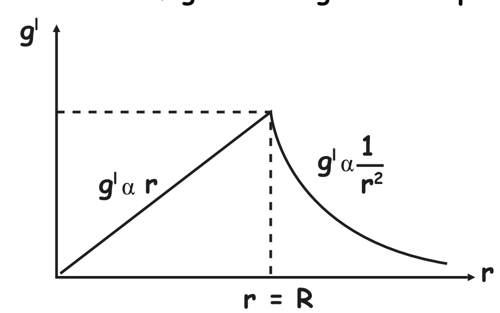
Absolute decrease $= \dfrac{\Delta g}{g} = g - g^l = \dfrac{dg}{R}$

Fractional decrease $= \dfrac{\Delta g}{g} = \dfrac{g-g^l}{g} = \dfrac{d}{R}$

Percentage decrease $= \dfrac{\Delta g}{g}\times100 = \dfrac{d}{R}\times100$

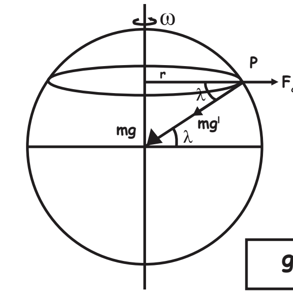
Very imp graph

The graphical representation of change in the value of g' with height and depth



for $r \le R$, $g^l = \dfrac{gr}{R}$ for $r \le R$, $g^l = \dfrac{gR^2}{r^2}$

● Variation of g due to rotation of earth



Latitude - Angle which the line joining the point to the centre of earth makes with the equatorial plane

$$\boxed{g^l = g - \omega^2 R\cos^2\lambda}$$

Note ⇒ value of $\omega^2 R = 0.034$

For poles $\lambda = 90°$ $g^l = g$

There is no effect of rotational motion of the earth on the value of g at poles.

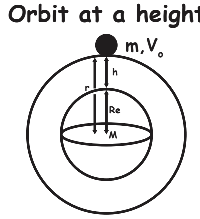
For equator $\lambda = 0°$ $g^l = g - \omega^2 R$

The effect of rotational motion of the earth on the value of g at the equator is maximum.

When a body of mass m is moved from equator to the poles, weight increases by an amount $m\,(g_p - g_e) = m\omega^2 R$

ORBITAL VELOCITY

Orbit at a height 'h' from the surface



$$V_o = \sqrt{\frac{GM}{(R+H)}} = \sqrt{\frac{gR^2}{(R+H)}}$$

$$V_o = \sqrt{\frac{GM}{R}} = \sqrt{gR}$$

If orbit is closer to earth's surface(neglect 'h') $V_o = \sqrt{\dfrac{GM}{R}} = \sqrt{gR}$

(called minimum orbit, velocity-first cosmic velocity)

Note - for easy calculations

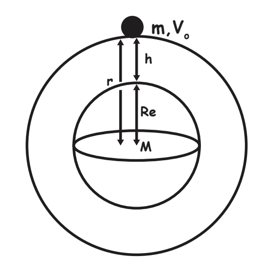
$\sqrt{gR} = 8$ km/s or $\sqrt{\dfrac{GM}{R}} = 8$ km/s $= 8\times 10^3$ m/s

or $\dfrac{GM}{R} = 64\times10^6$

KE, PE OR TE FOR AN ORBITING SATELLITE

$KE = \dfrac{GMm}{2r}$ $U = -\dfrac{GMm}{r}$

$TE = -\dfrac{GMm}{2r}$

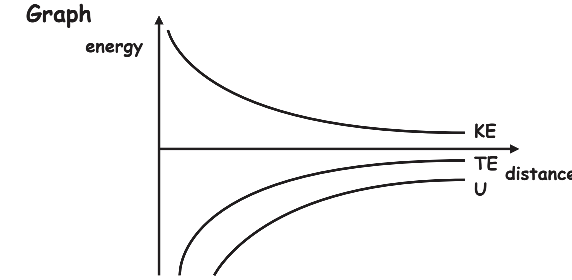


Relation KE, U & TE

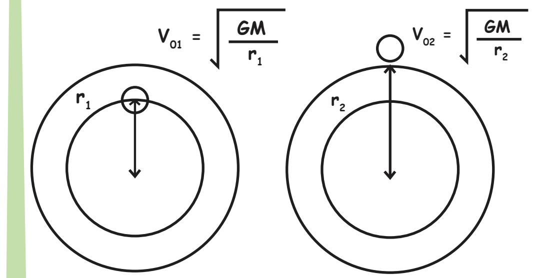
U = 2 x T.E

K.E = - T.E

Graph



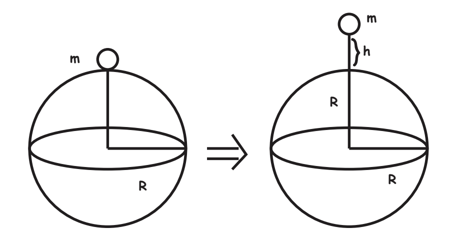
WORK DONE IN MOVING OBJECT FROM ONE ORBIT TO ANOTHER



CONCEPT - WORK DONE BY EXTERNAL AGENT = CHANGE IN MECHANICAL ENERGY

$$W = E_2 - E_1 = \frac{GMm}{2}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

WORK DONE IN MOVING OBJECT FROM SURFACE OF EARTH TO HEIGHT h ABOVE SURFACE



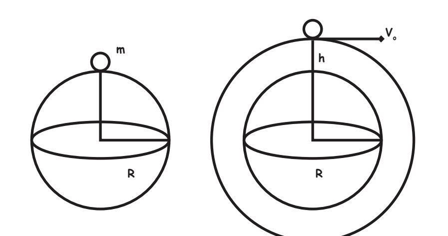
$$W = (U_f - U_i) = \frac{-GMm}{R+h} - \left[\frac{-GMm}{R}\right]$$

Or, $\boxed{W = mgh\,\dfrac{R}{R+h}}$

Work done to move object to a height h = R $\boxed{W = \dfrac{mgR}{2}}$

Work done to move object to a height h = R/2 $\boxed{W = \dfrac{mgR}{3}}$

WORK DONE IN MOVING OBJECT FROM SURFACE TO CIRCULAR ORBIT



$W = E_f - E_i$

$W = E_{total} - U_i$ $\quad W = \dfrac{-GMm}{2(R+h)} + \dfrac{GMm}{R}$

PHYSICS WALLAH